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Modular Degeneracy of Vacuum Zero Potential Point and Gauge Transformations

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This paper presents an extended gauge theory which can contain the Higgs mechanism, considering $\lambda\phi^4$ field theory as an example. It can introduce interaction into different vacuum modular degeneracy states and break modular degeneracy. At the same time we can obtain both massless and massive vector bosons. According to the extended gauge theory, gauge transformations can be classified into two kinds: those with fixed parameter, called simply definite gauge transformations, which have a function of breaking modular degeneracy, and indefinite gauge transformations, which have a function of keeping phase degeneracy.

1. INTRODUCTION

Since Yang–Mills gauge theory was introduced (Yang and Mills, 1954), unified theories of interactions have achieved brilliant success. Electroweak theory (Weinberg, 1967; Salam, 1968), grand unified theories, supersymmetry, and the gauge theory of gravitation were built one after another and were developed rapidly (Pati and Salam, 1973; Utiyama, 1956; Sohnius, 1985). BRS transformations in the quantization of the gauge theory of gravitation are also a further development of gauge theory ideas (Becchi *et al.*, 1975). Physicists are realizing more and more the significance of gauge theory.

However, gauge fields are massless; we have to depend on the Higgs mechanism in order to obtain massive gauge particles. If we extend the concepts of gauge transformations, it is easy to include the Higgs mechanism in gauge theories. There are two merits of this method: One is that the processes of producing mass become very concise. Gauge particles obtain mass directly from the extended gauge transformations; it is no longer neces-

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sary to seek help from the Higgs mechanism. The other is that conditions under which the gauge field produced is massive or massless are very clear. Here we give a distinct physical rationale in place of the extremely complex and even mysterious Higgs mechanism.

2. DEGENERACY, SYMMETRY, AND BREAKING

It is known that if the Lagrangian function of a system is invariant under a group of transformations, we say that the system has the symmetry of the group. For example, the electromagnetic interaction has the symmetry of the group $U(1)$. In physics, a certain kind of symmetry corresponds to degeneracy of the system. Under the symmetry operation, different states with the same Lagrangian function are degenerate. In the gauge theory of the group $U(1)$, the Lagrangian function is invariant under local phase transformation. The symmetry of a system shows a certain remaining degeneracy of the system.

Moreover, gauge transformations have another important task, that is, introducing interaction. The function of gauge transformations is to break some original degeneracy. In essence, therefore, generally, gauge transformations have two functions: breaking degeneracy and keeping degeneracy. The task of physics is not only to study the symmetry of interactions, but also to break the symmetry, and thereby to find new interactions.

In order to make the above discussion specific, we consider the following system:

$$\mathcal{L}_1 = (\partial_\mu\phi)^*(\partial_\mu\phi) + \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 \tag{1}$$

where the potential due to the ϕ term is

$$U(\phi) = \lambda(\phi^*\phi)^2 - \mu^2\phi^*\phi \tag{2}$$

Setting $U(\phi) = 0$, we obtain two field modular solutions for ϕ:

$$|\phi_1| = |c_1| = 0 \tag{3}$$

$$|\phi_2| = |c_2| = \frac{\mu}{\sqrt{\lambda}} \tag{4}$$

This shows that the vacuum zero potential has a degeneracy corresponding to the above two modular values of ϕ.

At the zero potential point, $\mathcal{L}_1$ can be expressed as

$$\mathcal{L}_1^{c_1} = (\partial_\mu\phi)^*(\partial_\mu\phi) - U(c_1) \tag{5}$$

or

$$\mathcal{L}_1^{c_1} = (\partial_\mu\phi)^*(\partial_\mu\phi) - U(c_2) \tag{6}$$

Since $U(c_1) = U(c_2) = 0$, the degeneracy of the vacuum zero potential implies that $\mathcal{L}_1$ is degenerate under the translation operation of ϕ according to $\phi \to \phi + c_i$ $(i = 1, 2)$. Generally, $\mathcal{L}_1$ is n-fold degenerate under the translation operation of ϕ according to $\phi \to \phi + c_i$ $(i = 1, 2, \ldots, n)$ for $U(c_i) = 0$.

Now we define transformations for the degenerate states (5) and (6) as follows: Consider a field near the vacuum zero potential point,

$$\phi = \frac{1}{\sqrt{2}}(\eta(x) + |c_i|)e^{i\alpha(x)} \tag{7a}$$

Make the transformation

$$\phi \to \phi' = e^{-i\alpha}\phi = \frac{1}{\sqrt{2}}(\eta + |c_i|) \tag{7b}$$

$$\partial_\mu \to D_\mu = \partial_\mu - iA_\mu \tag{7c}$$

The purpose of the transformation is to introduce the interaction. We call the transformations (7b) and (7c) definite gauge transformations, where c_i and $\alpha(x)$ are definite.

2.1. The Degenerate State for $|c_1| = 0$

We obtain a Lagrangian density after introducing new interactions

$$\mathcal{L}_2 = (\partial_\mu + iA_\mu)\frac{1}{\sqrt{2}}\eta(\partial_\mu - iA_\mu)\frac{1}{\sqrt{2}}\eta + \frac{1}{2}\mu^2\eta^2 - \frac{1}{4}\lambda\eta^4$$
$$- \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \tag{8}$$

Obviously, $\mathcal{L}_2 \neq \mathcal{L}_1$, and modular degeneracy is broken. We obtain a massless vector boson A_μ simultaneously. The degeneracy corresponding to phase is kept. Hence, $\mathcal{L}_2$ is invariant under the following transformations:

$$\phi \to \phi' = e^{-i\beta(x)}\phi \tag{9a}$$

$$D_\mu\phi \to D'_\mu\phi' = e^{-i\beta(x)}D_\mu\phi \tag{9b}$$

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu\beta \tag{9c}$$

We call the transformations (9) indefinite gauge transformations.

2.2. The Degenerate State for $|c_1| = \mu/\sqrt{\lambda}$

We obtain a Lagrangian density after introducing new interactions

$$\mathcal{L}_3 = (\partial_\mu + iA_\mu)\frac{1}{\sqrt{2}}\left(\eta + \frac{\mu}{\sqrt{\lambda}}\right)(\partial_\mu - iA_\mu)\frac{1}{\sqrt{2}}\left(\eta + \frac{\mu}{\sqrt{\lambda}}\right)^2$$
$$+ \frac{1}{2}\mu^2\left(\eta + \frac{\mu}{\sqrt{\lambda}}\right)^2 - \frac{1}{4}\lambda\left(\eta + \frac{\mu}{\sqrt{\lambda}}\right)^4 - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \quad (10)$$

Here the term $\frac{1}{2}(\mu^2/\lambda)B_\mu^2$ represents a vector boson of mass $\mu/\sqrt{\lambda}$. In addition, the scalar field $\eta(x)$ has mass $\sqrt{2}\mu$. The result is not different from that of the Higgs mechanism. It should be pointed out that the Higgs mechanism only breaks modular degeneracy and does not break phase degeneracy.

We see that it is worth replacing the Higgs mechanism by definite gauge transformations because, first, the Higgs mechanism cannot obtain massless vector bosons, otherwise definite gauge transformations could obtain both massive and massless vector bosons, and second, definite gauge transformations simplify greatly the complicated process of the Higgs mechanism, and the physics becomes clear.

3. EXTENDED GAUGE TRANSFORMATION THEORY

We show the role of degeneracy breaking by gauge transformations and its relation to the Higgs mechanism as follows:

$$\mathcal{L}_1 \xrightarrow{\text{Degeneracy of zero potential}} \begin{cases} \mathcal{L}_1^{c_1} \xrightarrow{\text{Gauge transformations}} \mathcal{L}_2 \xrightarrow{\text{Gauge transformations}} \mathcal{L}_2 \\ \mathcal{L}_1^{c_2} \xrightarrow[\text{(modular degeneracy is broken)}]{\text{Higgs mechanism}} \mathcal{L}_3 \xrightarrow[\text{(phase degeneracy is kept)}]{\text{Gauge transformations}} \mathcal{L}_3 \end{cases}$$

This shows that gauge transformations have two functions: breaking modular degeneracy of $\mathcal{L}_1$ and keeping phase degeneracy of $\mathcal{L}_1$. Furthermore, we see that the original gauge transformations break the degeneracy of $\mathcal{L}_1^{c_1}$, and, as a result, yield massless vector bosons; on the other hand, the Higgs mechanism breaks the degeneracy of $\mathcal{L}_1^{c_2}$ and yields massive vector bosons.

We call the degeneracy-breaking gauge transformations "definite gauge transformations"; the transformation parameter $|c_i|$ is the modular solution for $U(\phi) = 0$.

Appropriately, we call gauge transformations that keep degeneracy "indefinite gauge transformations." In this way, the extended gauge transformations are essentially

Extended gauge transformations

$$\begin{cases} \text{Definite gauge transformations} \\ \quad \text{(remove mode degeneracy)} \\ \text{Indefinite gauge transformations} \\ \quad \text{(retain phase degeneracy)} \end{cases}$$

According to this point of view of gauge transformations, the Higgs mechanism actually corresponds to definite gauge transformations whose parameter is $|c_2|$.

On the basis of the above framework, we include the Higgs mechanism in gauge transformation theories. At the same time the procedure becomes very simple.

The concepts of the extended gauge theory are perfectly suitable for Yang–Mills theory.

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